A High-Efficiency Post-Processing Method for Computing Magnetic Flux in Coils Considering Magnetic and Conductive Regions

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A high-efficiency post-processing method for computing the magnetic flux in coils is discussed and applied to magnetodynamic problem analysis. The method firstly separates the contribution of magnetic flux in a coil into three parts, respectively generated by source coils, magnetic regions and conductive regions. It can reduce the mesh size used around the coil, thus increasing the computational efficiency. It is well adapted to any numerical technique, such as the finite element and volume integral methods. A strong magnetic–electric coupling problem solved with volume integral method was used to validate the performance of the post-processing computational method.

***Index Terms*—Coils, magnetic flux, magnetodynamic, volume integral method (VIM).**

I. Introduction

MAGNETIC flux in coils is an important quantity in the analysis, design and optimization of electromagnetic devices. The magnetic flux in a coil is generally calculated in the post-processing step of computational electromagnetic problems, where the field distribution is firstly obtained by given numerical methods, such as the finite element method (FEM) or the volume integral method (VIM).

With integral formulation method, it is unnecessary to discretize the air region, such kind of approach is therefore more efficient for the study of electromagnetic device with predominant air regions. Thanks to the development of matrix compression algorithms, solving the electromagnetic problems by integral equations is becoming more and more popular and efficient, from magnetostatic applications [1]-[2] to magnetodynamic applications [3]-[4] in the presence of both magnetic and conductive volume regions, and also including a coupling with external circuits.

After solving the problem and getting all the field quantities on the mesh, it is necessary to put forward a high efficient computational method for evaluating magnetic flux in coils. This paper proposes a new general approach dividing the magnetic flux in a coil into the summation of three terms: one generated by source coils, one by magnetic region and one by conductive region. The computation of these three terms are discussed in the following part.

II. Magnetic Flux Computation

By magnetic vector potential $\mathbf{A}$ formulation, the magnetic flux through a coil k, with a region denoted by Ω_{c_k}, can be expressed by the integral

$$\Phi_k = \int_{\Omega_{0_k}} \mathbf{j}_{0_k} \cdot \mathbf{A}\, \mathrm{d}\Omega \tag{1}$$

where the $\mathbf{j}_{0_k}$ is a vector function describing the normalized current density of the considered coil region Ω_{0_k}. For a magnetodynamic problem containing: 1) source coil regions Ω_0, 2) non-conducting magnetic regions Ω_m and 3) electrical conductive regions Ω_c, as illustrated in Fig. 1, the magnetic vector potential $\mathbf{A}$ can be separated in three components, i.e $\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_\mathrm{m} + \mathbf{A}_\mathrm{c}$, where $\mathbf{A}_0$, $\mathbf{A}_\mathrm{m}$ and $\mathbf{A}_\mathrm{c}$ are generated respectively by the three aforementioned regions. The contribution of total magnetic flux Φ through a coil k can be therefore expressed by

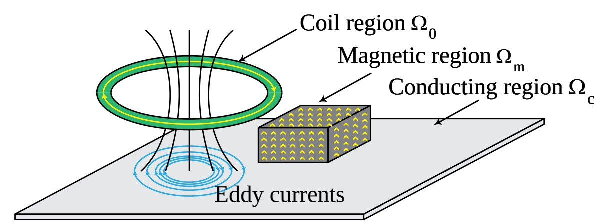

Fig. 1. Illustration of a magnetodynamic problem with coils, magnetic regions and conductive region.

$$\Phi_k = \Phi_{0_k} + \Phi_{\mathrm{m}_k} + \Phi_{\mathrm{c}_k} \tag{2}$$

where Φ_{0_k} is the contribution of all source coils in vacuum, Φ_{m_k} is the contribution of the magnetization in magnetic regions and Φ_{c_k} is the contribution of eddy currents in electrical conductive regions.

In a magnetodynamic problem with multiple coils, the component of magnetic vector potential $\mathbf{A}_0$ on the coil k, which is generated by all the coils in vacuum, can be analytically computed by

$$\mathbf{A}_0 = \frac{\mu_0}{4\pi} \sum_l \left(\int_{\Omega_{0_l}} \frac{\mathbf{j}_{0_l}}{r}\, \mathrm{d}\Omega \right) i_l \tag{3}$$

where the sum of l means all coils in the problem. Thus, the first part of flux Φ_{0_k} in (2), which is generated by all coils is the problem, can be calculated by

$$\Phi_{0_k} = \frac{\mu_0}{4\pi} \int_{\Omega_{0_k}} \mathbf{j}_{0_k} \cdot \left(\sum_l i_l \int_{\Omega_{0_l}} \frac{\mathbf{j}_{0_l}}{r}\, \mathrm{d}\Omega \right) \mathrm{d}\Omega \tag{4}$$

where the second integral on a coil region Ω_{0_l} can be calculated by a semi-analytical method [5].

Furthermore, in the case of integral formulations, the other two components of the magnetic vector potential, $\mathbf{A}_\mathrm{m}$ and $\mathbf{A}_\mathrm{c}$, can be obtained by integrating receptively on the magnetic region Ω_m and conductive region Ω_c:

$$\begin{cases} \mathbf{A}_{\mathrm{m}} = \dfrac{\mu_0}{4\pi}\displaystyle\int_{\Omega_{\mathrm{m}}} \mathbf{M}\times\nabla\left(\dfrac{1}{r}\right)\mathrm{d}\Omega_{\mathrm{m}} \\ \mathbf{A}_{\mathrm{c}} = \dfrac{\mu_0}{4\pi}\displaystyle\int_{\Omega_{\mathrm{c}}} \dfrac{\mathbf{J}}{r}\mathrm{d}\Omega_{\mathrm{c}} \end{cases} \tag{5}$$

Therefore, the other two flux components in (2) are calculated by the double integral, as:

$$\begin{cases} \Phi_{\mathrm{m}_k} = \dfrac{\mu_0}{4\pi}\displaystyle\int_{\Omega_{0_k}} \mathbf{j}_{0_k}\left[\int_{\Omega_{\mathrm{m}}} \mathbf{M}\times\nabla\left(\dfrac{1}{r}\right)\mathrm{d}\Omega_{\mathrm{m}}\right]\mathrm{d}\Omega \\ \Phi_{\mathrm{c}_k} = \dfrac{\mu_0}{4\pi}\displaystyle\int_{\Omega_{0_k}} \mathbf{j}_{0_k}\left[\int_{\Omega_{\mathrm{c}}} \dfrac{\mathbf{J}}{r}\mathrm{d}\Omega_{\mathrm{c}}\right]\mathrm{d}\Omega \end{cases} \tag{6}$$

In (6), the two double integrals are related to the considered coil region Ω_{0_k}, it should be thus sufficiently meshed to have a precise result. A great number of elements will increase dramatically the computation time, it is relatively inefficient to solve directly the double integral, especially dealing with the complex geometries.

A method which allows obtaining Φ_{m_k} by integrating directly in the magnetic region has been presented in [6]. It proposes a method to calculate the flux generated by the magnetization in magnetic region can be calculated by

$$\Phi_{\mathrm{m}_k} = \int_{\Omega_{0_k}} \mathbf{j}_{0_k}\cdot\mathbf{A}_{\mathrm{m}}\,\mathrm{d}\Omega = \mu_0\int_{\Omega_{\mathrm{m}}} \mathbf{h}_{0_k}\cdot\mathbf{M}\,\mathrm{d}\Omega \tag{7}$$

where $\mathbf{M}$ means the magnetization field in magnetic regions Ω_{m} and $\mathbf{h}_{0_k}$ represents the magnetic field generated by the coil k with 1 A. The computational method in (7) has shown a good efficacy in the study of inductive power transmission system for the mutual-inductance calculation. However, the method can only deal with the magnetostatic problems and has not been extended yet to conductive region in the magnetodynamic context.

Similarly, in order to increase the computational efficiency by solving directly the double integral in (6) for the flux component Φ_{c_k}, we tend to propose in this paper a new method which leads directly to a single integration in the conductive region Ω_{c_k}, as:

$$\Phi_{\mathrm{c}_k} = \int_{\Omega_{\mathrm{c}_k}} \mathbf{j}_{0_k}\cdot\mathbf{A}_{\mathrm{c}}\,\mathrm{d}\Omega = \int_{\Omega_{\mathrm{c}_k}} \mathbf{a}_{0_k}\cdot\mathbf{J}_{\mathrm{c}}\,\mathrm{d}\Omega \tag{8}$$

where $\mathbf{a}_{0_k}$ represents the normalized magnetic vector potential generated by the coil k with 1 A, and $\mathbf{J}_{\mathrm{c}}$ means the eddy current density, which can be easily calculated by $\mathbf{A}$ formulation that combines equivalent network based on Whitney facet interpolation and Green's function volume integral method proposed in [4].

The computation of the magnetic flux generate by eddy current is finally changed into a single integral which is related only to the domain of electrical conductive region Ω_{c}.

III. Magnetodynamic Application Example

The proposed post-processing method is tested by a strong magnetic–electric coupling problem, as illustrated in Fig. 2. The 3-D axisymmetric problem is composed of: a solenoid coil (radius 6 mm, thickness 0.1 mm, height 10 mm) and a linear magnetic core (average radius 4 mm, thickness 2 mm, height 10 mm, relative permeability = 100) inside. A thin surface copper region (radius 5.5 mm, thickness 0.1 mm, height 10 mm, conductivity 55×10^{6} S/m) is placed between the solenoid coil and the magnetic core to realize an electromagnetic shielding by eddy currents.

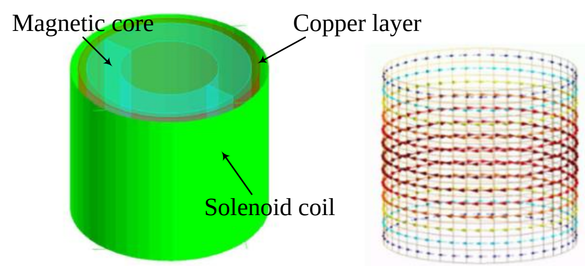

Fig. 2. A test example of strong magnetic–electric coupling problem. (a) Geometry and (b) eddy currents density on the conductive region.

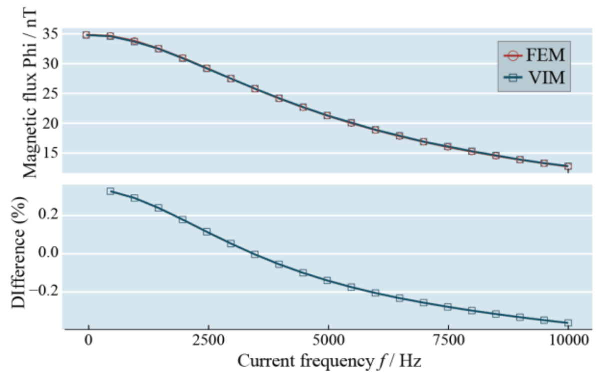

Fig. 3. Comparison of the magnetic flux in the coil obtained by FEM (ref) and VIM.

Figure 3 shows the variation of the flux in the coil versus the frequency, with an axisymmetric finite element analysis to the same problem in the software Flux as reference. First result is provided at 0 Hz without any eddy currents and the module of magnetic flux in the coil has reached the maximum value. When the frequency increases, eddy currents begin to act as a shield and the flux decreases. Moreover, the comparison to the results with a finite element method shows a difference less than 0.4% at any frequency.

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